AN ALPINE MEETING ON NONPOSITIVE CURVATURE IN KÄHLER GEOMETRY

AUSSOIS – 5-9 JUNE 2023

PROGRAM

	Monday 05/06/23	Tuesday 06/06/23	Wednesday 07/06/23	Thursday 08/06/23	Friday 09/06/23
9h - 10h	Stéphane DRUEL	Yohan BRUNEBARBE	Omprokash DAS	Benoît CLAUDON	Shigeharu TAKAYAMA
10h30-11h25	Bruno KLINGLER	Christian SCHNELL	Bruno KLINGLER	Christian SCHNELL	Sébastien BOUCKSOM
11h30-12h25	Christian SCHNELL	Bruno KLINGLER	Christian SCHNELL	Bruno KLINGLER	Valentino TOSATTI
14h - 15h	SHORT TALKS	Martin DERAUX		SHORT TALKS	
15h30 - 16h30	Benjamin BAKKER	Erwan ROUSSEAU		Carolina TAMBORINI	

Main speakers -

- Benjamin Bakker - *UIC*
- Sébastien Boucksom - *IMJ-PRG*
- Yohan Brunebarbe - *U. Bordeaux*
- Benoît Claudon - *U. Rennes 1*
- Omprokash Das - *TIFR Mumbai*
- Martin Deraux - *U. Grenoble Alpes*
- Stéphane Druel - *U. Lyon 1*
- Bruno Klingler - *Humboldt U. – Mini-course*
- Erwan Rousseau - *U. Bretagne Occidentale*
- Christian Schnell - *Stony Brook U. – Mini-course*
- Shigeharu Takayama - *U. Tokyo*
- Carolina Tamborini - *Utrecht U.*
- Valentino Tosatti - *NYU*

Short talks -

Monday -

- Rodolfo Aguilar Aguilar - *U. of Miami*
- Roberto Albesiano - *Stony Brook U.*
- Niklas Mueller - *U. Duisburg-Essen*

Thursday -

- Chung-Ming Pan - *U. Toulouse III*
- Angel David Ríos Ortiz - *Sapienza U. di Roma*
- William Sarem - *ENS Lyon*

*
* *

TITLES AND ABSTRACTS

ON DERIVATIVES OF PERIOD MAPS

Benjamin Bakker (University of Illinois at Chicago)

There are now a number of functional transcendence results for variations of Hodge structures. The Ax–Schanuel conjecture for period maps, for example, classifies atypical algebraic relations satisfied by the flag variety coordinates of the Hodge filtration with respect to a flat basis. There is also a version dealing directly with period integrals proven recently in joint work with J. Tsimerman. As the coordinates of the period map are rational functions of the period integrals, the latter is more general, and some information is lost in passing from period integrals to period maps. In this talk I will explain how to reverse this process: we show that in fact all period integrals can be recovered from the derivatives of the period map. Along the way we provide a more systematic treatment of the differential equations satisfied by the coordinates of the period map. This is joint work with J. Pila and J. Tsimerman.

KÄHLER METRICS OF CONSTANT SCALAR CURVATURE, K-STABILITY AND OPENNESS

Sébastien Boucksom (Sorbonne Université)

By a fundamental result of Chen-Cheng, a polarized projective manifold admits a (unique) constant scalar curvature Kähler metric iff the Mabuchi K-energy functional is coercive. According to the Yau-Tian-Donaldson conjecture, this should also be equivalent to uniform K-stability, an algebro-geometric condition that can be phrased as the coercivity of a non-Archimedean version of the Mabuchi functional. I will present a joint work with Mattias Jonsson, in which we show that coercivity is an open condition with respect to the polarization, in both the complex and non-Archimedean settings.

EXISTENCE OF THE SHAFAREVICH MORPHISM FOR SEMISIMPLE LOCAL SYSTEMS ON QUASI-PROJECTIVE VARIETIES

Yohan Brunebarbe (Université de Bordeaux)

In an attempt to understand which complex analytic spaces can be realised as the universal covering of a complex algebraic variety, Shafarevich asked whether the universal covering of any smooth projective variety X is holomorphically convex. In other words, does there exists a proper holomorphic map from the universal covering of X to a Stein analytic space? Although still open, Shafarevich question has been answered positively e.g. when the fundamental group of X is either virtually nilpotent (Katzarkov) or admits a faithful complex linear representation (Eyssidieux-Kaztarkov-Pantev-Ramachandran). Motivated by the generalization of Shafarevich

question to non-compact varieties, I will explain the following result (initially proved by Eyssidieux for smooth projective varieties): under a maximality assumption, the covering space of a normal connected complex algebraic variety associated to the kernel of a semisimple complex representation of its fundamental group admits a proper surjective holomorphic map with connected fibres onto a normal analytic space with no positive-dimensional compact analytic subspace.

NUMERICAL CHARACTERIZATION OF BALL AND TORUS QUOTIENTS

Benoît Claudon (Université de Rennes 1)

If X is a compact Kähler manifold whose first Chern class $c_1(X)$ is a non-positive multiple of a Kähler class, it is known (as a consequence of the existence of Kähler–Einstein metric by Aubin and Yau) that X satisfies the inequality $(2(n+1)c_2(X) - nc_1(X)^2) \cdot \alpha^{n-2} \geq 0$ where $\alpha = -c_1(X)$ if the latter is Kähler, or any Kähler class if $c_1(X) = 0$. Moreover, the equality case can be characterized in a geometric way: it happens exactly when X is covered by the unit ball $\mathbb{B}^n$ or by a complex torus T. It is then natural to look at the following generalization: is it possible to recognize a quotient $\mathbb{B}^n/\Gamma$ or T/G by a group acting merely properly discontinuously? This leads us to consider singular spaces, and even pairs (X, Δ) to encode the possible hypersurfaces having non-trivial stabilizers in $\mathbb{B}^n$ or T. Once the right notions of Chern numbers have been introduced, such quotients can be characterized in the very same way as in the smooth case. This is based on a joint work with Patrick Graf and Henri Guenancia.

GENERALIZED MMP FOR KÄHLER 3-FOLDS

Omprokash Das (TIFR Mumbai)

Generalized pairs is defined by Birkar and Zhang for projective varieties and have been playing increasingly important role in various applications of the Minimal Model Program. This led to a rapid progress in the development of the generalized MMP for projective varieties. In a joint work with Christopher Hacon and Jose Y. Yanez we show that the generalized MMP can be defined for Kähler varieties in more general situations than that of projective varieties. More specifically, we consider a pair $(X, B + T)$, where X is a normal analytic variety, B is a $\mathbb{Q} - divisor$ and T is a closed positive $(1,1)$-current which is a pushforward of a closed positive $(1,1)$ nef form from a higher model of X. We establish most of the standard results of the MMP in this context when X is a compact Kähler variety of dimension at most 3.

A SMOOTH COMPLEX HYPERBOLIC SURFACE WITH ONE END

Martin Deraux (Université Grenoble Alpes)

I will explain some techniques to construct explicit torsion-free subgroups in lattices in the isometry group of the complex hyperbolic plane. Applying this to lattices generated by three complex reflections, we get some groups with interesting properties, including the fundamental group of a tower of smooth non-compact complex hyperbolic surfaces of finite volume with a single end.

PROJECTIVELY FLAT LOG SMOOTH PAIRS

Stéphane Druel (Université Lyon 1)

In this talk, I will discuss projective log smooth pairs with numerically flat normalized logarithmic tangent bundle. Generalizing works of Jahnke-Radloff and Greb-Kebekus-Peternell, we show that, passing to an appropriate finite cover and up to isomorphism, these are the projective spaces or the log smooth pairs with numerically flat logarithmic tangent bundles blown-up at finitely many points away from the boundary. On the other hand, the structure of log smooth pairs with numerically flat logarithmic tangent bundle is well understood: they are toric fiber bundles over finite etale quotients of abelian varieties.

HODGE THEORY, BETWEEN ALGEBRAICITY AND TRANSCENDENCE (MINI-COURSE)

Bruno Klingler (Humboldt Universität)

Hodge theory, as developed by Deligne and Griffiths, is an essential tool for analyzing the geometry and arithmetic of complex algebraic varieties. It is a crucial fact that at heart, Hodge theory is NOT algebraic. On the other hand, according to both the Hodge conjecture and the Grothendieck period conjecture, this transcendence is severely constrained. This minicourse will illustrate this tension between algebraicity and transcendence by focusing on the study of Hodge loci, presenting in passing the necessary tools (functional transcendence, o-minimality, etc...).

AN ALBANESE CONSTRUCTION FOR CAMPANA'S C-PAIRS

Erwan Rousseau (Université de Bretagne Occidentale)

We will explain a construction of Albanese maps for orbifolds (or C-pairs) with applications to hyperbolicity such as a generalization of the Bloch-Ochiai theorem. (Joint with Stefan Kebekus).

DEGENERATING VARIATIONS OF HODGE STRUCTURE IN DIMENSION ONE (MINI-COURSE)

Christian Schnell (Stony Brook)

I will give four lectures about degenerating complex variations of Hodge structure in dimension one, covering parts of the paper with the same title that Claude Sabbah and I put on arXiv last June. The content of the four lectures is roughly going to be like this:

1. Introduction and examples,
2. Estimates for the Hodge norm,
3. Convergence of the Hodge filtration and the nilpotent orbit theorem,
4. The limiting mixed Hodge structure.

LIMITS OF L^p-STRUCTURES ON PLURICANONICAL SYSTEMS

Shigeharu Takayama (University of Tokyo)

We study limits of the canonical L^p-space structures on pluricanonical systems on compact complex manifolds. We construct a canonical mixed L^p-space structure on the central fiber of a one parameter degeneration. We prove that the canonical L^p-spaces on smooth nearby fibers converge to the mixed L^p-space.

SYMMETRIC SPACES AND GEOMETRY OF THE TORELLI LOCUS

Carolina Tamborini (Universiteit Utrecht)

Riemannian symmetric spaces are Riemannian manifolds with special symmetry properties. They are important in various fields of geometry. In algebraic geometry, they appear naturally as spaces parametrizing certain Hodge structures. In particular, the moduli space A_g of principally polarized abelian varieties is a quotient of the Siegel space, which is, in fact, a (hermitian) symmetric space (of the non-compact type). In the talk, we consider the Torelli locus. This is the closure in A_g of the image of the moduli space M_g of smooth, complex algebraic curves of genus g via the Torelli map $j : M_g \longrightarrow A_g$. We describe the problem of studying the (local) geometry of the Torelli locus in A_g and its relation with symmetric subspaces of the Siegel space. Also, we explain how this is linked to a conjecture by Coleman and Oort.

THE KÄHLER CONE OF CALABI-YAU MANIFOLDS

Valentino Tosatti (New York University)

The Kähler cone of a compact Kähler manifold is an open convex cone in a finite-dimensional real vector space which parametrizes the cohomology classes of Kähler metrics. A very natural problem is to understand the nature of its boundary, and whether cohomology classes on the boundary have representatives which have geometric significance. I will discuss in detail the case of Calabi-Yau manifolds, where these boundary classes can be probed using Ricci-flat metrics, and describe some recent results and open questions.

*
* *

SHORT TALKS

ON NILPOTENT QUOTIENTS OF NORMAL QUASI-PROJECTIVE GROUPS

Rodolfo Aguilar Aguilar (University of Miami)

We show that if X is a normal complex quasi-projective variety, the quasi-Albanese map of which is proper, then the torsion-free nilpotent quotients of $\pi_1(X)$ are, up to a controlled finite index, the same ones as those of the normalisation of its quasi-Albanese image. In the normal quasi-projective case, the étale Galois cover of X associated to the nilpotent completion of $\pi_1(X)$ is thus holomorphically convex. Joint work with F. Campana.

A DEFORMATION APPROACH TO SKODA'S DIVISION THEOREM

Roberto Albesiano (Stony Brook)

In 1972 H. Skoda proved a version of the effective Nullstellensatz. More precisely, let $h_1, \dots, h_r$ be fixed holomorphic sections of $E^* \otimes G \to X$, where E, G are holomorphic line bundles over a Stein manifold X. Given a holomorphic section g of G, is it always possible to write g as a linear combination $f_1 \otimes h_1 + \dots + f_r \otimes h_r$ for sections $f_1, \dots, f_r$ of E? Skoda's Division Theorem addresses this question and gives L2 bounds on the optimal solution $(f_1, \dots, f_r)$. In this talk I will explain a new proof of a Skoda-type theorem inspired by a deformation argument of B. Berndtsson and L. Lempert. In particular, we will see how to obtain L2 bounds on the optimal solution by carefully deforming a weight on the space of all linear combinations $f_1 \otimes v_1 + \dots + f_r \otimes v_r$ to single out the linear combination $f_1 \otimes h_1 + \dots + f_r \otimes h_r$ we are interested in.

TWO NON-VANISHING PROBLEMS CONCERNING THE ANTI-CANONICAL BUNDLE

Niklas Mueller (Universität Duisburg-Essen)

Recall that the famous Non-Vanishing Conjecture predicts that the (log) canonical bundle of a projective variety admits sections as soon as it is nef. We consider the dual question whether nefness of the anti-(log) canonical bundle implies the existence of sections. We answer this question affirmatively for threefold, many fourfold or assuming a generalisation of the classical Non-Vanishing Conjecture. The main ingredient is a new equivariant Non-Vanishing statement asserting that if the anti-(log) canonical bundle has sections then it automatically has sections which are invariant under the action of any fixed torus acting on the variety.

KÄHLER-EINSTEIN METRICS ON FAMILIES OF FANO VARIETIES

Chung-Ming Pan (U. Toulouse III)

In this talk, I will introduce a pluripotential approach to study uniform a priori estimates of Kähler-Einstein (KE) metrics on families of Fano varieties. After briefly recalling the basic tools and the variational approach, I will define a notion of convergence of quasi-plurisubharmonic functions in families of normal varieties and extend several classical properties under this context. Finally, I will explain how these elements lead to an analytic proof of the openness of existing singular KE metrics and a uniform estimate of KE potentials. This is joint work with Antonio Trusiani.

ASYMPTOTIC BASE LOCI FOR HYPERKAHLER MANIFOLDS

Angel David Ríos Ortiz (Sapienza Università di Roma)

I will present joint work with Francesco Denisi (Bologna) about properties of the asymptotic base loci on projective hyperkahler manifolds. More specifically, we will give a numerical characterization of the asymptotic base loci of big divisors on hyperkahler manifolds and study how they vary while moving a big divisor class in the big cone, using the divisorial Zariski decomposition, and the Beauville-Bogomolov-Fujiki form.

CURVATURE PROPERTIES FOR TOROIDAL COMPACTIFICATIONS OF BALL QUOTIENTS

William Sarem (ENS Lyon)

In this talk, we will describe toroidal compactifications of ball quotients, which are examples of compact Kähler manifolds. Some of them admit a Riemannian metric with nonpositive sectional curvature and we showed that they admit no Kähler metric with nonpositive sectional curvature (in dimension 2, this was proved by Di Cerbo). They also provide the first example of a compact complex manifold admitting a Kähler metric with quasi-negative holomorphic bisectional curvature, but admitting no Kähler metric with negative holomorphic (bi)sectional curvature. This answers a question of Diverio on the existence of such manifolds.
